STOCHASTIC PROGRAMMING WITH APPLICATIONS IN ECONOMICS, FINANCE AND INSURANCE

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Review of results and publications

1. Stochastic programming. An important role in the development of stochastic programming was played by the seminal work of R. T. Rockafellar and R. J.-B. Wets [5,6] and E. B. Dynkin [1-3] in the 1970s. This work served as the starting point for the line of research reviewed below. Most of the results were motivated by economic applications.

[1] Dynkin, E. B.: Some probability models for a developing economy, Soviet Mathematics Doklady 200 (1971) 523-525.
[2] Dynkin, E. B.: Stochastic concave dynamic programming, USSR Mathematics Sbornik 16 (1972) 501-515.[1]
[3] Dynkin, E. B. and Yushkevich, A. A.: Controlled Markov processes and their applications, Springer, N. Y., 1979. DOI.
[4] Rockafellar R.T. and Wets R.J.-B.: Stochastic convex programming: Kuhn-Tucker conditions, Journal of Mathematical Economics, 2 (1975) 349-370. DOI.
[5] Rockafellar R.T. and Wets R.J.-B.: Nonanticipativity and L_1-martingales in stochastic optimization problems, Stochastic Systems: Modeling, Identification and Optimization II, R.J.-B. Wets, ed., Math. Programming Study, v. 6, 1976, North Holland, pp. 170-187. DOI

2. Stochastic programming and economic growth over an infinite time horizon. Paper [6] extends Dynkin's results [1-3] to an infinite time horizon and stationary stochastic models. Stationarity is defined in terms of ergodic theory: via a time shift – a measure-preserving transformation of the underlying probability space. For stationary models, infinite sums of utilities typically diverge, therefore a central role in the paper is played by the Ramsey-Weizsäcker "overtaking" optimality criterion. The key mathematical result in [6] is an $\langle L_\infty, L_1 \rangle$ version of the Kuhn-Tucker theorem for a stationary stochastic programming problem defined in terms of a measure-preserving transformation of the probability space [6, Section 4]. The proof of the existence of Lagrange multipliers/dual variables in L_1 is based on the Yosida-Hewitt theorem.

[6] Evstigneev I. V.: Optimal stochastic programs and their supporting prices, 1974, in: Mathematical Models in Economics, Amsterdam, J. Los and M. Los, eds. North Holland, 219-252. PDF.

Results in [6] serve as the basis for Chapters 4 and 5 in the book [7].

[7] Arkin V. I. and Evstigneev I. V.: Stochastic models of control and economic dynamics, Academic Press, London, 1987. PDF

[1] The main results of [2] are presented in Chapter 9 of the book [3].

3. Lagrange multipliers for stochastic programming problems. Paper [8] took a look at the classical results of Rockafellar and Wets [4,5] from a different angle, providing a direct construction of Lagrange multipliers via a Kuhn-Tucker theorem in $\langle L_\infty, L_1 \rangle$, without the use of the apparatus of convex duality.

[8] Evstigneev I. V.: Lagrange multipliers for the problems of stochastic programming, 1976, Lect. Notes Econ. Math. Syst., n. 133, 34-48. PDF

Article [9] in Encyclopedia of Optimization provided a brief review of results of this kind, and paper [10] applied them to the theory of stochastic growth.

[9] Evstigneev I. V. and Flåm S. D.: Stochastic programming: Non-anticipativity and Lagrange multipliers, in: Encyclopedia of Optimization, Kluwer Academic Publishers, Dordrecht, 2001, v. 4, 332-338. DOI
[10] Evstigneev I. V. and Flåm S. D.: Rapid growth paths in multivalued dynamical systems generated by homogeneous convex stochastic operators, 1998, Set-Valued Analysis, v. 6, 61-82 DOI

4. Non-convex stochastic programming. Paper [11] developed the Bellman principle of dynamic programming in the context of finite-stage non-convex stochastic optimization problems.

[11] Evstigneev I. V.: Measurable selection and dynamic programming, Math. Oper. Res., 1 (1976) 52-55. PDF

5. Regular conditional expectations. For the purposes of [11] techniques of regular conditional expectations (RCE) of random variables depending on parameters were developed. This is a device which makes it possible to work in a general situation, when conventional conditional distributions (regular conditional probabilities) do not exist. RCE and their extensions to set-valued random variables were studied in [12-14].

[12] Dynkin E. B. and Evstigneev I. V.: Regular conditional expectations of correspondences, Teor. Ver. i Primen.(Theory of Probab. and Appl.) 21 (1976) 333-347. PDF
[13] Evstigneev I. V.: Regular conditional expectations of random variables depending on parameters, 1986, Teor. Ver. i Primen. (Theory of Probab. and Appl.) 31 (1986) 586-589. PDF
[14] Evstigneev I. V.: Regular conditional expectations and the continuum hypothesis, in: The Dynkin Festschrift, M. I. Freidlin, ed., Birkhäuser, Boston, 1994, 85-93. DOI

5. Equilibrium stochastic programming. All the above results pertaining to the convex case were obtained in $\langle L_\infty, L_1 \rangle$ settings and relied upon the Yosida-Hewitt theorem. An alternative approach using the “biting lemma” was proposed in:

[15] Evstigneev I. V. and Flåm S. D.: Convex stochastic duality and the "biting lemma", Journal of Convex Analysis, 9 (2002) 237-244. PDF

This was needed to pass from the conventional stochastic optimization models to more general and complex stochastic equilibrium models. Here, roughly speaking, the objective function depends not only on the primal, but also on the dual variables. This is characteristic for economic models, where the dual variables represent prices. It turned out that in such contexts, the Yosida-Hewitt theorem does not work, but the biting lemma does. This was demonstrated in [16].

[16] Evstigneev I. V. and Taksar M. I.: Equilibrium states of random economies with locally interacting agents and solutions to stochastic variational inequalities in $<L_\infty,L_1>$, Annals of Operations Research, Special Issue "Stochastic Equilibrium Problems in Economics and Game Theory", I. V. Evstigneev, S. D. Flåm, L. J. Mirman, Eds., 114 (2002) 145-165.). DOI

The work in this direction with the corresponding references is described in more detail in the review: Equilibrium, growth and local interactions.

6. Stochastic programming on directed graphs. Papers [17, 18] deal with stochastic optimization and control problems on directed graphs.

[17] Evstigneev I. V.: Controlled random fields on a directed graph, Teor. Ver. i Primen. (Theory of Probab. and Appl.), 33 (1988) 465-479. PDF
[18] Evstigneev I. V. and Taksar M. I.: Convex stochastic optimization for random fields on graphs: A method of constructing Lagrange multipliers, Mathematical Methods of Operations Research, 54 (2001) 217-237. DOI

7. Stochastic programming, insurance and information. Paper [19] develops a non-traditional approach to insurance modelling and a theory of economic information distinct from the classical Shannon's information theory. A key role is played by the Rockafellar-Wets [5] prices of information constraints.

[19] Evstigneev I. V., Klein Haneveld W. K. and Mirman, L. J.: Robust insurance mechanisms and the shadow prices of information constraints, Journal of Applied Mathematics and Decision Sciences, 3 (1999) 85-128. PDF