

# VON NEUMANN-GALE DYNAMICAL SYSTEMS WITH APPLICATIONS IN ECONOMICS AND FINANCE

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Joint work with [E. Babaei](#), [W. Bahsoun](#), [S.D. Flåm](#), [S.A. Pirogov](#), [K.R. Schenk-Hoppé](#), and [M.V. Zhitlukhin](#)

Von Neumann-Gale dynamical systems are defined in terms of multivalued operators possessing properties of convexity and homogeneity. These operators assign to each element of a given cone a convex subset of the cone describing possible one-step transitions from one state of the system to another. The classical, deterministic theory of such dynamics was originally aimed at the modelling of economic growth (von Neumann 1937 and Gale 1956). Key results on von Neumann-Gale dynamical systems may be regarded as multivalued nonlinear versions of the Perron-Frobenius theorem on eigenvectors and eigenvalues of positive matrices.

Attempts to build a stochastic generalization of this theory were undertaken in the 1970s by Dynkin, Radner and their research groups. However, the initial attack on the problem left many questions unanswered. Substantial progress was made only in the late 1990s, and final solutions to the main open problems were obtained only in the last 10-15 years.

At about the same time it was observed that stochastic analogues of von Neumann-Gale dynamical systems provide a natural and convenient framework for financial modelling (asset pricing and hedging under transaction costs). This observation not only gave a new momentum to studies in the field and posed new interesting questions, but also made it possible to find a key to the solution of old problems. The new, financial interpretation of the mathematical notions and objects at hand amazingly suggested the way of proofs that could not be found earlier.

## Working papers

Von Neumann-Gale model, market frictions, and capital growth, [SSRN Working Paper 3314852](#), January 2019 (with [E. Babaei](#), [K.R. Schenk-Hoppé](#) and [M.V. Zhitlukhin](#)).

Log-optimal and rapid paths in von Neumann-Gale dynamical systems, Economics Discussion Paper [EDP-1902](#), The University of Manchester, January 2019 (with [E. Babaei](#) and [K.R. Schenk-Hoppé](#)).

Von Neumann-Gale dynamics and capital growth in financial markets with frictions, Economics Discussion Paper [EDP-1815](#), The University of Manchester, November 2018 (with [E. Babaei](#), [K.R. Schenk-Hoppé](#) and [M.V. Zhitlukhin](#)).

A multidimensional Fatou lemma for conditional expectations, [arXiv:1811.00640 \[math.OA\]](#), November 2018 (with [E. Babaei](#) and [K.R. Schenk-Hoppé](#)).

## Publications

Growth-optimal investments and numeraire portfolios under transaction costs, 2013, in: Handbook of the Fundamentals of Financial Decision Making ( [L.C. MacLean](#) and [W.T.](#)

[Ziemba](#), eds.), World Scientific, Singapore, pp. 789-808 (with [W. Bahsoun](#) and M.I. Taksar).  
[DOI](#)

Rapid paths in von Neumann-Gale dynamical systems, 2008, *Stochastics*, v. 80, 129-142 (with [W. Bahsoun](#) and M.I. Taksar.) [DOI](#)

Stochastic equilibria in von Neumann-Gale dynamical systems, 2008, *Transactions of the American Mathematical Society*, v. 360, 3345-3364 (with [K.R. Schenk-Hoppé](#)). [DOI](#)

Pure and randomized equilibria in the stochastic von Neumann-Gale model, 2007, *Journal of Mathematical Economics*, v. 43, 871-887 (with [K.R. Schenk-Hoppé](#)). [DOI](#)

Asset pricing and hedging in financial markets with transaction costs: An approach based on the von Neumann-Gale model, 2006, *Annals of Finance*, v. 2, 327-355 (with M.A.H. Dempster and M.I. Taksar). [DOI](#)

The von Neumann-Gale growth model and its stochastic generalization, 2006, in: *Handbook on Optimal Growth*, R.-A. Dana, C. Le Van, T. Mitra and K. Nishimura, eds., Springer, New York, 337-383 (with [K.R. Schenk-Hoppé](#)). [DOI](#)

Rapid growth paths in convex-valued random dynamical systems, 2001, *Stochastics and Dynamics*, v. 1, 493-509 (with M.I. Taksar). [DOI](#)

Turnpike theorems for positive multivalued stochastic operators, 2000, *Advances in Mathematical Economics*, v. 2, 1-20 (with S. Anoulova and V.M. Gundlach). [DOI](#)

Convex-valued random dynamical systems: A variational principle for equilibrium states, 1999, *Random Operators and Stochastic Equations*, v. 7, 23-38 (with L. Arnold and V.M. Gundlach).  
[DOI](#)

Rapid growth paths in multivalued dynamical systems generated by homogeneous convex stochastic operators, 1998, *Set-Valued Analysis*, v. 6, 61-82 (with [S.D. Flåm](#)). [DOI](#)

Homogeneous convex models in the theory of controlled random processes, 1980, *Doklady AN SSSR (Soviet Math. Dokl.)*, v. 253, n. 3, 524-527. [PDF](#)