Evolutionary Behavioural Finance

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The models considered in this field combine elements of stochastic dynamic games (strategic frameworks) and evolutionary game theory (solution concepts).
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- In its classical version, this theory assumes that market participants act so as to **maximize utilities** of consumption subject to budget constraints.
- It is assumed that the objectives of economic agents can be described in terms of well-defined and precisely stated constrained optimization problems.
The goal of the present study is to develop an alternative equilibrium concept – **behavioural equilibrium**, admitting that market actors may have different patterns of behaviour determined by their individual psychology, which are **not necessarily describable in terms of individual utility maximization**.
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Behavioural equilibrium

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**Objectives** might be of an **evolutionary nature**: **survival** (especially in crisis environments), **domination** in a market segment, fastest capital **growth**, etc. They might be **relative** – taking into account the performance of the others.
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- **Behavioural finance**: Shiller (the 2013 Nobel Prize in Economics) and others.
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- **Behavioural finance:** Shiller (the 2013 Nobel Prize in Economics) and others.
- **Evolutionary game theory:** J. Maynard Smith and G. R. Price (1973)
Basic Model

- Invest in assets

Portfolio: $x_i(t) = (x_i(t), 1, ..., x_i(t), K)_{R^K+}$

Vector of market prices: $p(t) = (p(t), 1, ..., p(t), K)_{R^K+}$

The value of the portfolio: $h_p(t) = \sum_{k=1}^{K} p(t), k x_i(t), k$

Stochastic process of states of the world: $a_1, a_2, ...$

History of the process: $a(t) = (a_1, ..., a_t)$

Total amount of asset $k$ in period $t$: $V_t, k(a_t) > 0$

Dividend of asset $k$ in period $t$: $D_t, k(a_t) \geq 0$

Vector of investment proportions selected by trader $i$: $\lambda_{it} = (\lambda_{it}, 1, ..., \lambda_{it}, K)_{R^K+}$

$\lambda_{it} = \lambda_{it}(a_t)$

$\lambda_{it}^2 \Delta K$, $\Delta K = f(c_1, ..., c_K)_{R^K+}$

$\sum_{k=1}^{K} c_k = 1$
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- $\lambda_t^i \in \Delta^K, \Delta^K = \{(c_1, ..., c_K) \in \mathbb{R}_+^K : c_1 + ... + c_K = 1\}$ (action of $i$)
Strategic framework

- **Strategy (portfolio rule)** of investor $i$: a rule

\[ \lambda_t^i = \Lambda_t^i(a^t, H_t) \]

prescribing what vector $\lambda_t^i$ of investment proportions to select at each time $t$ depending on the history $a^t = (a_1, ..., a_t)$ of states of the world and the history of play

\[ H_t = \{ \lambda_s^i : s < t, \ i = 1, ..., I \}. \]
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$$H_t = \{\lambda_s^i : s < t, \; i = 1, ..., I\}.$$ 

Basic strategy: $\Lambda_t^i = \Lambda_t^i(a^t)$ depends only on $a^t$ and not on $H_t$. 
Short-run equilibrium

- Short-run equilibrium:

\[ p_{t,k} V_{t,k} = \alpha \sum_{i=1}^{l} \lambda_{t,k}^i \langle p_t + D_t, x_{t-1}^i \rangle \]  

(1)
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- investor \( i \)'s wealth: \( w_{t}^{i} = \langle p_{t} + D_{t}, x_{t-1}^{i} \rangle \)
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\[ \alpha \] is the investment rate, and \( 0 < \alpha < 1 \)
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- investor \( i \)'s portfolio \( x_t^i = (x_{t,1}^i, ..., x_{t,K}^i) \):

\[ x_{t,k}^i = \frac{\alpha \lambda_{t,k}^i \langle p_t + D_t, x_{t-1}^i \rangle}{p_{t,k}} = \frac{\alpha \lambda_{t,k}^i w_t^i}{p_{t,k}} \]
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- Equations (1) can be written as

\[ V_{t,k} = \sum_{i=1}^{l} x_{t,k}^i \text{ (supply = demand)} \]
Dynamics. Outcome of the game

- Fix a strategy profile $\Lambda = (\Lambda^1, ..., \Lambda^I)$ of $I$ investors. Generate step by step, from $t$ to $t + 1$ equilibrium asset price vectors $p_t$ and for each investor $i$, vectors of investment proportions $\lambda^i_t$, and portfolios $x^i_t$. 

Outcome of the game for player $i$ is the random sequence of $i$’s market shares $r^i_0, r^i_1, r^i_2, ...$. 
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- Compute for each $t, i$ investor $i$’s wealth $w^i_t = \langle p_t + D_t, x^i_{t-1} \rangle$, the total market wealth $W_t := \sum_{i=1}^I w^i_t$ and investors’ market shares $r^i_t := w^i_t / W_t$. 
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- **Outcome** of the game for player $i$ is the random sequence of $i$’s market shares $r^i_0, r^i_1, r^i_2, ...$. 
Solution concept: Survival strategy

A strategy $\Lambda^i$ of player $i$ is called a **survival strategy** if for any strategies $\Lambda^j$ of players $j \neq i$ the market share $r^i_t$ of player $i$ is bounded away from zero almost surely:

$$\inf_t r^i_t > 0 \text{ almost surely.}$$
Central Results

- **Assumption 1.** For all $t, k$ with strictly positive probability, $E_tD_{t+s,k} > 0$ for some $s \geq 1$. 
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- **Assumption 2.** $V_{t,k} = \gamma^t V_k$, $\gamma \geq 1$. 

- **Theorem 1.** The portfolio rule $\Lambda_t = (\lambda_t)$ is a survival strategy.

- **Theorem 2.** If $\Lambda_t = (\lambda_t)$ is a basic survival strategy, then $\sum_{t=0}^{\infty} \lambda_t < \infty \ (a.s.)$.

- Theorems 1 and 2: existence and asymptotic uniqueness of survival strategy.
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- Define: \( \rho = \alpha / \gamma \) and \( R_{t,k} = D_{t,k} V_k / \sum_{m=1}^{K} D_{t,m} V_m \) (relative dividends).
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- Consider the basic portfolio rule $\Lambda^* = (\lambda^*_t)$, where

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\lambda^*_{t,k} = E_t \sum_{l=1}^{\infty} (1 - \rho) \rho^{l-1} R_{t+l,k}
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- **Theorem 1.** The portfolio rule $\Lambda^*$ is a survival strategy.

- **Theorem 2.** If $\Lambda = (\lambda_t)$ is a basic survival strategy, then
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  \sum_{t=0}^{\infty} ||\lambda^*_t - \lambda_t||^2 < \infty \text{ (a.s.)}
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- **Theorems 1 and 2:** existence and asymptotic uniqueness of survival strategy.
Some References

This textbook is an elementary introduction to the key topics in mathematical finance and financial economics - two realms of ideas that substantially overlap but are often treated separately from each other. Our goal is to present the highlights in the field, with the emphasis on the financial and economic content of the models, concepts and results. The book provides a novel, unified treatment of the subject by deriving each topic from common fundamental principles and showing the interrelations between the key themes. Although the presentation is fully rigorous, with some rare and clearly marked exceptions, the book restricts itself to the use of only elementary mathematical concepts and techniques. No advanced mathematics (such as stochastic calculus) is used.