Evolutionary Behavioural Finance

Rabah Amir (University of Iowa) Igor Evstigneev (University of Manchester) Thorsten Hens (University of Zurich) Klaus Reiner Schenk-Hoppé (University of Manchester)

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- The models considered in this field combine elements of stochastic dynamic games (strategic frameworks) and evolutionary game theory (solution concepts).

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- In its classical version, this theory assumes that market participants act so as to **maximize utilities** of consumption subject to budget constraints.
- It is assumed that the objectives of economic agents can be described in terms of well-defined and precisely stated constrained optimization problems.

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- **Strategies** may involve, for example, mimicking, satisficing, rules of thumb based on experience, etc. Strategies might be **interactive** – depending on the behaviour of the others.
- **Objectives** might be of an **evolutionary** nature: **survival** (especially in crisis environments), **domination** in a market segment, fastest capital **growth**, etc. They might be **relative** – taking into account the performance of the others.

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- **Behavioural finance:** Shiller (the 2013 Nobel Prize in Economics) and others.
- **Evolutionary game theory**: J. Maynard Smith and G. R. Price (1973)

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- $\lambda_t^i \in \Delta^K$, $\Delta^K = \{(c_1, ..., c_K) \in \mathbb{R}_+^K : c_1 + ... + c_K = 1\}$ (action of i)

Strategic framework

- **Strategy (portfolio rule)** of investor i: a rule

$$\lambda_t^i = \Lambda_t^i(a^t, H_t)$$

prescribing what vector λ_t^i of investment proportions to select at each time t depending on the history $a^t = (a_1, ..., a_t)$ of states of the world and the history of play

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- **Basic** strategy: $\Lambda_t^i = \Lambda_t^i(a^t)$ depends only on a^t and not on H_t.

Short-run equilibrium

- Short-run equilibrium:

$$p_{t,k} V_{t,k} = \alpha \sum_{i=1}^{I} \lambda^{i}_{t,k} \langle p_t + D_t, x^{i}_{t-1} \rangle \qquad (1)$$

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- Equations (1) can be written as

$$V_{t,k} = \sum_{i=1}^{I} x^i_{t,k} \text{ (supply = demand)}$$

Dynamics. Outcome of the game

- Fix a strategy profile $\Lambda = (\Lambda^1, ..., \Lambda^I)$ of I investors. Generate step by step, from t to $t+1$ equilibrium asset price vectors p_t and for each investor i, vectors of investment proportions λ_t^i, and portfolios x_t^i.

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- **Outcome** of the game for player i is the random sequence of i's market shares $r_0^i, r_1^i, r_2^i, \ldots$.

Solution concept: Survival strategy

A strategy Λ^i of player i is called a **survival strategy** if for any strategies Λ^j of players $j \neq i$ the market share r_t^i of player i is bounded away from zero almost surely:

$$\inf_t r_t^i > 0 \text{ almost surely.}$$

Central Results

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- **Theorems 1 and 2**: existence and asymptotic uniqueness of survival strategy.

Some References

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