

Springer Texts in Business and Economics

Igor Evstigneev
Thorsten Hens
Klaus Reiner Schenk-Hoppé

Mathematical Financial Economics

A Basic Introduction

 Springer

Igor V. Evstigneev • Thorsten Hens •
Klaus Reiner Schenk-Hoppé

Mathematical Financial Economics

A Basic Introduction

 Springer

Igor V. Evstigneev
Economics, School of Social Sciences
University of Manchester
Manchester
United Kingdom

Thorsten Hens
Department of Banking and Finance
University of Zurich
Zurich
Switzerland

Klaus Reiner Schenk-Hoppé
Economics, School of Social Sciences
University of Manchester
Manchester
United Kingdom

ISSN 2192-4333 ISSN 2192-4341 (electronic)
Springer Texts in Business and Economics
ISBN 978-3-319-16570-7 ISBN 978-3-319-16571-4 (eBook)
DOI 10.1007/978-3-319-16571-4

Library of Congress Control Number: 2015939236

Springer Cham Heidelberg New York Dordrecht London
© Springer International Publishing Switzerland 2015

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media
(www.springer.com)

Preface



Tyche
Goddess of Chance and Fortune
By Tatjana Heinz

This textbook is a basic introduction to the key topics in mathematical finance and financial economics—two realms of ideas that substantially overlap but are often treated separately from each other. Our goal is to present the highlights in the field, with the emphasis on the financial and economic content of the models, concepts and results. The book provides a novel, unified treatment of the subject by deriving each topic from common fundamental principles and showing the interrelations between the key themes.

Although our presentation is fully rigorous, with some rare and clearly marked exceptions, we restrict ourselves to the use of only elementary mathematical concepts and techniques. No advanced mathematics (such as stochastic calculus) is used. The main source for the book, and a “proving ground” for testing our presentation of the material, are courses on mathematical finance, financial

economics and risk management which we have delivered, over the last decade, to undergraduate and graduate students in economics and finance at the Universities of Manchester, Zurich and Leeds.

The textbook contains 18 chapters corresponding to 18 lectures in a course based upon it. There are three chapters with problems and exercises, most of which have been used in tutorials, take-home tests and examinations, with full and detailed answers. The problems and exercises contain not only numerical examples, but also theoretical questions that complement the material presented in the body of the textbook. Two mathematical appendices provide rigorous definitions of some of the mathematical notions and statements of general theorems used in the text.

The textbook covers the classical topics, such as mean-variance portfolio analysis (Markowitz, CAPM, factor models, the Ross-Huberman APT), derivative securities pricing, and general equilibrium models of asset markets (Arrow, Debreu and Radner). A less standard but very important topic, which to our knowledge has not previously been covered in elementary textbooks, is capital growth theory (Kelly, Breiman, Cover and others). Absolutely new material, reflecting research achievements of recent years, is an introduction to new dynamic equilibrium models of financial markets combining behavioral and evolutionary principles.

A characteristic feature of financial economics is that it has to focus on the analysis of random, unpredictable market situations. To model such situations our discipline created powerful theoretical tools based on probability and stochastic processes. However, the power of human mind is not unlimited, and it can never fully eliminate the influence of chance and fortune, personified by goddess Tyche, looking at us from the epigraph to this book.

Manchester, UK
Zurich, Switzerland
Manchester, UK

Igor V. Evstigneev
Thorsten Hens
Klaus Reiner Schenk-Hoppé

Contents

Part I Mean-Variance Portfolio Analysis

1	Portfolio Selection: Introductory Comments	3
1.1	Asset Prices and Returns	3
1.2	Investor's Portfolio: Long and Short Positions	4
1.3	Return on a Portfolio	5
1.4	Mathematical Notation	8
2	Mean-Variance Portfolio Analysis: The Markowitz Model	11
2.1	Basic Notions	11
2.2	Optimization Problem: Formulation and Discussion	13
2.3	Assumptions	15
2.4	Efficient Portfolios and Efficient Frontier	16
3	Solution to the Markowitz Optimization Problem	19
3.1	Statement of the Main Result	19
3.2	Discussion	21
3.3	Proof of the Main Result	23
4	Properties of Efficient Portfolios	27
4.1	Mean and Variance of the Return on an Efficient Portfolio	27
4.2	Description of the Efficient Frontier	29
4.3	A Fund Separation Theorem	30
5	The Markowitz Model with a Risk-Free Asset	33
5.1	Data of the Model	33
5.2	Portfolio Optimization with a Risk-Free Asset	36
5.3	Solution to the Portfolio Selection Problem	38
6	Efficient Portfolios in a Market with a Risk-Free Asset	43
6.1	Expectations and Variances of Portfolio Returns	43
6.2	Efficient Frontier and the Capital Market Line	44
6.3	Tangency Portfolio	46
6.4	A Mutual Fund Theorem	50

7	Capital Asset Pricing Model (CAPM)	53
7.1	A General Result	53
7.2	An Equilibrium Approach to the CAPM	55
7.3	The Sharpe-Lintner-Mossin Formula	59
8	CAPM Continued	61
8.1	Security Market Line and the Pricing Formula	61
8.2	CAPM as a Factor Model	62
8.3	Applying Theory to Practice: Sharpe's and Jensen's Tests	64
9	Factor Models and the Ross-Huberman APT	69
9.1	Single- and Multi-Factor Models	69
9.2	Exact Factor Pricing	71
9.3	Ross-Huberman APT: Model Description	76
9.4	Formulation and Proof of the Main Result	78
10	Problems and Exercises I	83
 Part II Derivative Securities Pricing		
11	Dynamic Securities Market Model	105
11.1	Multi-Period Model of an Asset Market	105
11.2	Basic Securities and Derivative Securities	108
11.3	No-Arbitrage Pricing: Main Result	110
11.4	The No-Arbitrage Hypothesis and Net Present Value	112
12	Risk-Neutral Pricing	115
12.1	Risk-Neutral Measures	115
12.2	Fundamental Theorem of Asset Pricing	117
12.3	Asset Pricing in Complete Markets	119
13	The Cox–Ross–Rubinstein Binomial Model	125
13.1	The Structure of the Model	125
13.2	Completeness of the Model	127
13.3	Constructing a Risk-Neutral Measure	129
13.4	Examples	132
14	American Derivative Securities	137
14.1	The Notion of an American Derivative Security	137
14.2	Risk-Neutral Pricing of American Derivative Securities	139
14.3	The Pricing Algorithm	142
15	From Binomial Model to Black–Scholes Formula	145
15.1	Drift and Volatility	145
15.2	Modelling the Price Process	146
15.3	Binomial Approximation of the Price Process	147
15.4	Derivation of the Black–Scholes Formula	150
16	Problems and Exercises II	157

Part III Growth and Equilibrium

17 Capital Growth Theory 169
 17.1 Growth-Optimal Investments 169
 17.2 Strategies in Terms of Investment Proportions 171
 17.3 Results for Simple Strategies 173

18 Capital Growth Theory: Continued 177
 18.1 Log-Optimal Strategies 177
 18.2 Growth-Optimal and Numeraire Strategies 179
 18.3 Growth-Optimality for General Strategies 180
 18.4 Volatility-Induced Growth 183

19 General Equilibrium Analysis of Financial Markets 187
 19.1 Walrasian Equilibrium 187
 19.2 On the Existence of Equilibrium 190
 19.3 Rational Expectations and Equilibrium Pricing 192
 19.4 Arbitrage and Equilibrium 194

20 Behavioral Equilibrium and Evolutionary Dynamics 197
 20.1 A Behavioral Evolutionary Perspective 197
 20.2 Survival Strategies 201
 20.3 Links to the Classical Theory 203

21 Problems and Exercises III 205

Mathematical Appendices

A Facts from Linear Algebra 215

B Convexity and Optimization 219

Sources 223